Practice Problems 3

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[1] Sloan and Lorant (1977) studied the relationship between the length of time patients wait in a physicians office and certain demand and cost factors. They obtained data on typical patient waiting times for 4,500 physicians and reported a mean waiting time of 24.7 min and a standard deviation of 19.3 min.

Suppose a pediatrician does not have this set of data and has one of the nurses in the office monitor the waiting times for 64 randomly selected patients during the year.

a) What is the probability that the sample mean falling between 18 and 26 min?

b) Would your answer in part (a) change if Sloan and Lorant (1977) further showed that the date was exponentially distributed? Explain.

[2] A pharmaceutical manufacturer is concerned that the impurity concentration in pills should not exceed 3%. It is known that from a particular production run impurity concentrations follow a normal distribution. A random sample of 64 pills from a production run was checked, and the sample mean purity concentration was found to be 3.07%, and standard deviation is found to be 0.45%.

a) In the context of this problem, discuss whether one-sided or two-sided alternative hypothesis is more appropriate.

b) Test at the 5% level the appropriate (non-degenerate) null and alternative hypothesis.

c) Find the p-value of this test.

d) Suppose that the alternative hypothesis had been two-sided, rather than one-sided, with the null hypothesis $H_0 : \mu = 3$. State, without doing the calculations, whether the p-value of the test would be higher than, lower than, or the same as that found in part (c). Sketch a graph to illustrate your reasoning.

[3] The manufacturer of a new product claims that his product will increase output per machine by at least 29 units per hour. A line manager adopts the product on 9 of his machines, and finds that the average increase was only 26 with a standard deviation of 6.2.

a) Formulate the appropriate null and alternative hypotheses to test whether there is sufficient evidence to doubt the manufacturer's claim?

b) Calculate the appropriate test statistic, and then conclude your test at 95% confidence level.

c) Calculate the appropriate p-value? Is the result in the previous part statistically significant at the usual levels; namely 0.01, 0.05, and 0.10?

d) Now suppose that instead of 26, the sample mean is 31, how would you test the manufacturer's claim now? Just state the the appropriate null and alternative hypotheses.

[4] In a test of the Atkins weight loss program, 41 individuals participated in a randomized trial with overweight adults. After 12 months, the mean weight loss was found to be 2.1 lb, with a standard deviation of 4.8 lb.

a) What is the best point estimate for the mean weight loss of all overweight adults who follow the Atkins program?

b) Construct a 90% confidence interval for the mean weight loss for all such subjects?

c) Test the hypothesis that the mean weight loss is 3.2 lb.

d) In part (c), could we conclude the test simply by looking at the confidence interval constructed in part (b)? Explain. Also, what is the minimum mean weight loss that would be rejected by the sample data?

e) Suppose that a weight loss program is considered effective only if the weight loss is at least 3.2 lb after 12 months. Do you think, the Atkinson program seems to effective at 90% confidence level?

f) For part (e), could we use the confidence interval constructed in part (b)?

[5] **[SW 3.10]** Suppose a new standardized test is given to 100 randomly selected third-grade students in New Jersey. The sample average score $\overline{Y}$ on the test is 58 points, and the sample standard deviation, s_Y, is 8 points.

a) The authors plan to administer the test to all third-grade students in New Jersey. Construct a 95% confidence interval for the mean score of all New Jersey third graders.

b) Suppose the same test is given to 200 randomly selected third graders from Iowa, producing a sample average of 62 points and sample standard deviation of 11 points. Construct a 90% confidence interval for the difference in mean scores between Iowa and New Jersey.

c) Can you conclude with a high degree of confidence that the population means for Iowa and New Jersey students are different? (What is the standard error of the difference in the two sample means? What is the p-value of the test of no difference in means versus some difference?)

[6] **[SW 3.13]** Data on fifth-grade test scores (reading and mathematics) for 420 school districts in California yield $\overline{Y} = 646.2$ and standard deviation $s_Y = 19.5$.

a) Construct a 95% confidence interval for the mean test score in the population.

b) When the districts were divided into districts with small classes (< 20 students per teacher) and large classes (≥ 20 students per teacher), the following results were found:

Class Size	**Average Score**	**Standard Deviation**	n
Small	657.4	19.4	238
Large	650.0	17.9	182

Is there statistically significant evidence that the districts with smaller classes have higher average test scores? Explain.

[7] **[SW 3.16]** Grades on a standardized test are known to have a mean of 1000 for students in the United States. The test is administered to 453 randomly selected students in Florida; in this sample, the mean is 1013, and the standard deviation (s) is 108.

a) Construct a 95% confidence interval for the average test score for students in Florida.

b) Is there statistically significant evidence that students in Florida perform differently from other students in the United States?

[8] Suppose annual returns on two stocks, R_1 and R_2, are normally distributed with the following distributions:

$$R_1 \sim N(0.1, 0.2^2) \quad \text{and} \quad R_2 \sim N(0.2, 0.4^2)$$

The covariance between the two stock returns is $Cov(R_1, R_2) = 0.03$. Consider an equal weighted portfolio with return

$$R_p = 0.5R_1 + 0.5R_2$$

a) What are the mean and variance of R_p [i.e. $E(R_p)$ and $Var(R_p)$].

b) What is the distribution of R_p?

c) Find the probability that weighted portfolio return will be higher than 0.329, i.e. $P(R_p > 0.329) =?$

d) Suppose that there is another stock, R_3, that has the following normally distributed annual return and covariance with the second stock:

$$R_3 \sim N(0.1, 0.2^2) \qquad Cov(R_2, R_3) = -0.01$$

Would you replace R_1 with R_3 in the portfolio? Why?