Problem Set 7

Statistics - NYU, Summer 2016
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Section 1

[1] Consider the function

$$f(x) = \begin{cases} 1 - \frac{x}{2}, & 0 \le x \le 2 \\ 0, & \text{for all other x} \end{cases}$$

a) Show that $f(x)$ is a probability density function and plot $f(x)$.

b) Find $P(1/2 < X < 2/3)$?

c) Find the cumulative distribution function $F(x)$ without computing any integral? *(F(x) is the total area under $f(x)$ from 0 to x. So the graph of f(x) and the triangle formula are enough to find $F(x)$ in this example)*

d) Verify that $P(1/2 < X < 2/3) = F(2/3) - F(1/2)$.

[2] Consider the same function as in problem 1

$$f(x) = \begin{cases} 1 - \frac{x}{2}, & 0 \le x \le 2 \\ 0, & \text{for all other x} \end{cases}$$

a) Compute $E(X)$

b) For $g(X) = 3X^2 - 4X + 7$, compute $E[g(X)]$.

c) Compute $Var(X)$.

[3] Suppose that you pick five numbers at random from the interval (0,1). Assume that the numbers are independent. What is the probability that all numbers are greater than 0.7?

[4] A charitable organization solicits donations by telephone. Employees are paid $60 plus 20% of the money their calls generate each week. The amount of money generated in a week can be viewed as a random variable with a mean of $700 and a standard deviation of $130. Find the mean and standard deviation of an employees total pay in a week.

[**5**] A company services home air conditioners. It is known that times for service calls follow a normal distribution with a mean of 60 minutes and a standard deviation of 10 minutes.

   a) What is the probability that a single service call takes more than 65 minutes?

   b) Find the probability that a single service call takes between 50 and 70 minutes?

   c) The probability is 0.25 that a single service call takes more than how many minutes?

   d) Find the shortest range of times that includes 50% of all service calls.

   e) A random sample of four service calls is taken. What is the probability that exactly two of them take more than 65 minutes?

[**6**] A motel will have 80 rooms available on a certain night. Since, on average, 10% of the people making reservations do not show up, the motel makes 90 reservations for this night. What is the probability that there will be enough rooms for all who do show up? Use the normal approximation.

[**7**] A computer programmer has decided to use the exponential distribution to evaluate the reliability of a computer program. After 10 programming errors were found, the time (measured in days) to find the next error was determined to be exponentially distributed such that the mean time required to find the 11th error was 4 days.

   a) Graph this distribution.

   b) Find the probability that it will take more than 5 days to find the 11th error.

   c) Find the probability that it will take between 3 and 10 days to find the 11th error.

[**8**] Financial Managers Inc. buys and sells a large number of stocks routinely for the various accounts that it manages. Portfolio manager Andrea Colson has asked for your assistance in the analysis of the Johnson Fund. A portion of this portfolio consists of 10 shares of stock A and 8 shares of stock B. The price of A has a mean of 10 and a variance of 16, while the price of B has a mean of 12 and a variance of 9. The correlation between prices is 0.3.

   a) What are the mean and the variance of the portfolio value?

   b) Andrea has been asked to reduce the variance (risk) of the portfolio. She offers to trade 10 shares of stock A and receives two offers from which she can select one: 10 shares of stock 1 with a mean price of 10, a variance of 25, and a correlation with the price of stock B equal to -0.2; 10 shares of stock 2 with a mean price of 10, a variance of 9, and a correlation with the price of stock B equal to 0.5. Which offer should she select?

[9] Swapna David is a customer assistant consultant for ABC Information Systems, who provides assistance for computer users. The mean number of calls per hour is 40 from across the United States and calls are independent. She has just answered a call and is scheduled to take the next call. What is the probability that she will have at least 3 minutes to get a cup of tea before the next call?

[10] In a large department store a customer complaints office handles an average of six complaints per hour about the quality of service.

   a) What is the probability that in any hour exactly six complaints will be received?

   b) What is the probability that in any hour at most six complaints will be received?

   c) What is the probability that more than 20 minutes will elapse between successive complaints?

   d) What is the probability that fewer than 5 minutes will elapse between successive complaints?

Section 2

[11] Consider the function

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 \le x \le 2 \\ 0, & \text{for all other x} \end{cases}$$

   a) Show that $f(x)$ is a probability density function and plot $f(x)$. (To show f is a p.d.f. you can either evaluate the relevant integral or use simple geometry here)

   b) Verify that $P(1/2 < X < 3/2) = 3/4$?

   c) Find the cumulative distribution function $F(x)$ without computing any integral? *(F(x) is the total area under $f(x)$ from 0 to x. So the graph of f(x) and the triangle formula are enough to find $F(x)$ in this example)*

   d) Verify that $P(1/2 < X < 3/2) = F(2/3) - F(1/2)$ and interpret this in terms of areas.

[12] Consider the function

$$f(x) = \begin{cases} \frac{x^3}{5000}(10 - x), & 0 \le x \le 10 \\ 0, & \text{for all other x} \end{cases}$$

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a) Show that $f(x)$ is a probability density function.

b) Find $P(1 < X < 4)$?

c) Find $P(X > 6)$?

[13] The profit for a production process is equal to $2,000 minus two times the number of units produced. The mean and variance for the number of units produced are 500 and 900, respectively. Find the mean and variance of the profit.

[14] It is estimated that the time that a well-known rock band, the Living Ingrates, spends on stage at its concerts follow a normal distribution with a mean of 200 minutes and a standard deviation of 20 minutes.

a) What proportion of concerts played by this band lasts between 180 and 200 minutes?

b) An audience member smuggles a tape recorder into a Living Ingrates concert. The reel-to-reel tapes have a capacity of 245 minutes. What is the probability that this capacity will be sufficient to record the entire concert?

c) If the standard deviation of concert time was only 15 minutes, state, without doing any calculation, whether the probability that a concert would last more than 245 minutes would be larger than, smaller than, or the same as that found in part (b). Explain briefly.

d) The probability is 0.1 that a Living Ingrates concert will last less than how many minutes? (Assume, as originally, that the population standard deviation is 20 minutes.)

[15] Under fairly plausible assumptions about the behavior of clerks at supermarket check-out counters, it is possible to show that the time T (in minutes) a customer spends at a check-out counter is a random variable with the exponential distribution. Suppose a supermarket check-out counter has a mean number of customers per minute is equal to 1/3; that is, $\lambda = 1/3$ per minute. What is the probability that the length of time between a pair of customer arrivals is less than 6 min?

[16] Let the random variable X follow a normal distribution with mean $\mu = 48$ and variance $\sigma^2 = 60.84$ (i.e. $X \sim (48, 60.84)$).

a) Find the probability that X is greater than 58.

b) Find the probability that X is greater than 36 and less than 60.

c) The probability is 0.2 that X is greater than what number?

d) The probability is approximately 0.05 that X is in the symmetric interval about the mean between which two numbers?

e) For $Y = 2X + 14$, find the probability that Y is less than 110.