Problem Set 5

Statistics - NYU, Summer 2016
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Section 1

[1] Consider an urn containing 4 white and 2 black balls. Construct the probability distribution table for the following random variables:

    a) X = # of black balls in 2 draws **with** replacement

    b) Y = # of black balls in 2 draws **without** replacement

    c) Z = # of black balls in 20 draws **with** replacement

[2] For the random variables X, Y and Z defined in the previous problem calculate

    a) $E(X)$, $V(X) =$?

    b) $E(Y)$, $V(Y) =$?

    c) $E(Z)$, $V(Z) =$?

[3] A contractor estimates the probabilities for the number of days required to complete a certain type of construction project as follows:

Time (days)	1	2	3	4	5
Probability	0.05	0.2	0.35	0.3	0.1

    a) What is the probability that a randomly chosen project takes less than 3 days to complete?

    b) Find the expected time to complete the project.

    c) Find the standard deviation of time required to complete the project.

    d) Suppose the contractor's project cost is made of two parts: a fixed cost of $20,000 and $2,000 for each day taken to complete the project. Find the mean and standard deviation of total project cost.

    e) If three projects are undertaken, what is the probability that at least two of them will take at least 4 days to complete? Assume that project completion times are independent.

[4] It is estimated that 55% of the freshmen entering a particular college will graduate from that college in four years.

    a) For a random sample of 5 entering freshmen, what is the probability that exactly 3 will graduate in four years. What probability model should be used and explain why?

    b) For a random sample of 5 entering freshmen, what is the probability that a majority will graduate in four years.

    c) 80 entering freshmen are chosen at random. Find the mean and standard deviation of these 80 that will graduate in four years.

[5] From past experience, it is known 90% of one-year-old children can distinguish their mother's voice from the voice of a similar sounding female. A random sample of 20 one-year-olds is given this voice recognition test.

    a) Find the probability at least 3 children do not recognize their mother's voice.

    b) Find the probability all 20 children recognize their mother's voice.

    c) Let the random variable X denote the number of children who do not recognize their mother's voice. Find the mean of X.

    d) Let the random variable X denote the number of children who do not recognize their mother's voice. Find the variance of X.

    e) Find the probability that at most 4 children do not recognize their mother's voice?

[6] A military detection system is designed to warn a country of an enemy attack. A reliability question is whether a detection system will be able to identify an attack. Assume that a particular detection system has a .90 probability of detecting a missile attack, independent of its success in the previous missile attacks.

    a) What is the probability that the radar system will identify only three missile attacks out of ten missile attacks?

    b) On average how many missile attack you expect successfully to be detected out of 200 missile attacks?

    c) Now suppose that two detection systems are installed in the same area and operate independently. What is the probability that at least one of the systems will detect the attack?

    d) What is the probability that the new radar system with two detectors will identify at least 8 missile attacks out of ten missile attacks?

Section 2

[**7**] A long-distance taxi service owns four vehicles. These are of different ages and have different repair records. The probabilities that, on any given day, each vehicle will be available for use are 0.95, 0.90, 0.90, and 0.80. Whether one vehicle is available is independent of whether any other vehicle is available.

    a) Find the probability distribution for the number of vehicles available for use on a given day.

    b) Find the expected number of vehicles available for use on a given day.

    c) Find the standard deviation of the number of vehicles available for use on a given day.

[**8**] The World Series of baseball is to be played by team A and team B. The first team to win four games wins the series. Suppose that team A is the better team, in the sense that the probability is 0.6 that team A will win any specific game. Assume also that the result of any game is independent of that of any other.

    a) What is the probability that team A will win the series?

    b) What is the probability that a seventh game will be needed to determine the winner?

[**9**] A notebook computer dealer mounts a new promotional campaign. Purchasers of new computers may, if dissatisfied for any reason, return them within 2 days of purchase and receive a full refund. The cost to the dealer of such a refund is $100. The dealer estimates that 15% of all purchasers will, indeed, return computers and obtain refunds. Suppose that 50 computers are purchased during the campaign period.

    a) What is the probability that at most three computers will be refunded?

    b) Find the mean and standard deviation of the number of these computers that will be returned for refunds.

    c) Find the mean and standard deviation of the total refund costs that will accrue as a result of these 50 purchases.

[**10**] Consider an urn containing 4 white and 3 black balls. Construct the probability distribution table for the following random variables:

    a) X = # of black balls in 2 draws **with** replacement

b) Y = # of black balls in 2 draws **without** replacement

For the rest of the problem, suppose the urn also contains 2 green balls (i.e. 4W, 3B, 2G) and again construct the probability distribution table for the following random variables:

c) Z = # of black or white balls in 2 draws **with** replacement

d) T = # of black balls in 2 draws **without** replacement

[**11**] For the random variables X, Y and Z defined in the previous problem calculate

a) $E(X)$, $V(X) =$?

b) $E(Y)$, $V(Y) =$?

c) $E(Z)$, $V(Z) =$?