Problem Set 4

Statistics - NYU, Summer 2016
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Section 1

[1] There are three red chips and two blue chips in a bowl. The red chips are numbered 1, 2, and 3, respectively, and the blue chips are numbered 1 and 2, respectively. Suppose two chips are to be drawn at random and without replacement. *(Recall that in class, we found the sample space for this experiment, see the slides for Ch. 3)*

a) Find the probability that these chips have the same number.

b) Find the probability that these chips have the same color.

c) Find the probability that these chips have either the same number or the same color.

[2] A bowl contains 16 chips, of which 6 are red, 7 are white, and 3 are blue. Suppose four chips are to be drawn at random and without replacement. *(See PS3-7)*

a) Find the probability that each of the four chips is red.

b) Find the probability that none of the four chips is red.

c) Find the probability that there is at least one chip of each color.

[3] A coin is to be tossed until a head appears twice in a row. If the coin is fair, what is the probability that it will be tossed exactly four times?

[4] Given $P(A_1) = 0.4$, $P(B_1|A_1) = .6$, and $P(B_1|A_2) = .7$, calculate $P(A_1|B_1)$?

[5] In a recent survey about capital punishment, 64% of the respondents said that they support capital punishment. Females comprised 48% of the sample, and of the females, 46% supported capital punishment.

a) What is the probability that a randomly selected person is a female and capital punishment supporter?

b) What is the probability that a randomly selected person is a male and does support capital punishment?

c) What is the probability that a randomly selected person is a male and does not support capital punishment?

d) Suppose we select a person who does not support capital punishment, what is the probability that the person is male?

[6] In a large corporation, 80% of the employees are men and 20% are women. The highest levels of education obtained by the employees are graduate training for 10% of men, undergraduate training for 30% of men, and high school training for 60% of the men. The highest levels of education obtained by the employees are graduate training for 15% of women, undergraduate training for 40% of women, and high school training for 45% of the women.

a) What is the probability that a randomly chosen employee will be a man with only a high school degree?

b) What is the probability that a randomly chosen employee will have graduate training?

c) What is the probability that a randomly chosen employee who has graduate training is a man?

d) Are gender and level of education of employees in this corporation statistically independent?

e) What is the probability that a randomly chosen employee has not had graduate training is a woman?

[7] You are responsible for detecting the source of the error when the computer system fails. From your analysis you know that 50% of errors are disk drive errors, 30% are computer memory errors, and the remainder are operating system errors. From the component performance standards you know that, when a disk drive error occurs, the probability of failure is 0.60; when a computer memory error occurs, the probability of failure is 0.70; and when an operating system error occurs, the probability of failure is 0.3. Given the information from the component performance standards, what is the probability of a disk drive error, given that a failure occurred?

[8] Alice and Beatriz go target shooting together. Both shoot at a target at the same time. Suppose Alice hits the target with probability 0.7, whereas Beatriz, independently, hits the target with probability 0.4.

a) Given that the target is hit, what is the probability that Beatriz hit it?

b) Given that exactly one shot hit the target, what is the probability that it was Beatriz's shot?

Section 2

[9] A dean has found that 62% of entering freshmen and 78% of community college transfers eventually graduate. Of all entering students, 73% are freshmen and the remainder are community college transfers.

a) What is the probability that a randomly chosen entering student is a freshmen who will eventually graduate?

b) Find the probability that a randomly chosen entering student will eventually graduate?

c) What is the probability that a randomly chosen entering student either is freshmen or will eventually graduate (or both)?

d) Are the events "eventually graduates" and "enters as community college transfer" statistically independent?

[10] A restaurant manager classifies customers as regular, occasional, or new, and finds that all customers 50%, 40%, and 10%, respectively, fall into these categories. The manager found that wine was ordered by 70% of regular customers, by 50% of the occasional, and by 30% of the new customers.

a) What is the probability that a randomly chosen customer orders wine?

b) If wine is ordered, what is the probability that the person ordering is a regular customer?

c) If wine is ordered, what is the probability that the person ordering is an occasional customer?

[11] Of 100 patients with a certain disease, 10 were chosen at random to undergo a drug treatment that increases the cure rate from 50% for those not given the treatment to 75% for those given the drug treatment.

a) What is the probability that a randomly chosen patient both was cured and was given the drug treatment?

b) What is the probability that a patient who was cured had been given the drug treatment?

c) What is the probability that a specific group of 10 patients was chosen to undergo the drug treatment? (Leave your answer in terms of factorials.)

[12] A market-research group specializes in providing assessments of the prospects of sites for new childrens toy stores in shopping centers. The group assesses prospects as good, fair, or poor. The records of assessments made by this group were examined, and it was found that for all stores that had annual sales over $1,000,000, the assessments were good for 70%, fair for 20%, and poor for 10%. For all stores that turned out to be unsuccessful, the assessments were good for 20%, fair for 30%, and poor for 50%. It is known that 60% of new clothing stores are successful and 40% are unsuccessful.

a) For a randomly chosen store, what is the probability that prospects will be assessed as good?

b) If prospects for a store are assessed as good, what is the probability that it will be successful?

c) Are the events prospects assessed as good and store is successful statistically independent?

d) Suppose that five stores are chosen at random. What is the probability that at least one of them will be successful?

[13] Robert Smith uses either regular plowing or minimal plowing to prepare the cornfields on his Minnesota farm. Regular plowing was used for 40% of the field acreage. Analysis after the crop was harvested showed that 50% of the high-yield acres were from minimal-plowing fields and 40% of the low yield fields were from fields with regular plowing. What is the probability of a high yield if regular plowing is used? What is the probability that a field with high yield had been prepared using regular plowing?

[14] Staff, Inc., a management consulting company, is surveying the personnel of Acme Ltd. It determined that 35% of the analysts have an MBA and that 40% of all analysts are over age 35. Further, of those who have an MBA, 30% are over age 35.

a) What is the probability that a randomly chosen analyst both has an MBA and also is over age 35?

b) What is the probability that a randomly chosen analyst who is over age 35 has an MBA?

c) What is the probability that a randomly chosen analyst has an MBA or is over age 35?

d) What is the probability that a randomly chosen analyst who is over age 35 does not have an MBA?

e) Are the events MBA and over age 35 independent?

f) Are the events MBA and over age 35 mutually exclusive?

g) Are the events MBA and over age 35 collectively exhaustive?