Midterm 1

Statistics - NYU, Summer 2016
Ercan Karadas

[1] A college library has five copies of a certain text on reserve. Two copies (1 and 2) are first printings, and the other three (3, 4, and 5) are second printings. Consider the following experiment: *A student examines these books in random order, stopping only when a second printing has been selected.*

a) List the outcomes in the sample space. (5pt)

b) Let A denote the event that exactly one book must be examined. What outcomes are in A? (5pt)

c) Let B be the event that book 5 is the one selected. What outcomes are in B and find $P(B)$? (5pt)

d) Let C be the event that book 1 is not examined. What outcomes are in C and find $P(C)$? (10pt)

[2] A box in a certain supply room contains four 40-W lightbulbs, five 60-W bulbs, and six 75-W bulbs. Suppose that three bulbs are randomly selected.

a) What is the probability that exactly two of the selected bulbs are rated 75 W? (5pt)

b) What is the probability that all three of the selected bulbs have the same rating? (5pt)

c) What is the probability that one bulb of each type is selected? (5pt)

d) Suppose now that bulbs are to be selected one by one until a 75-W bulb is found. What is the probability that it is necessary to examine at least six bulbs? (10pt)

[3] Fifteen telephones have just been received at an authorized service center. Five of these telephones are cellular (CL), five are cordless (CS), and the other five are corded (CD) phones. Suppose that these components are randomly allocated the numbers 1, 2, . . ., 15 to establish the order in which they will be serviced.

a) In how many different ways the numbers can be allocated? (5pt)

b) What is the probability that all the cordless phones are among the first ten to be serviced? (5pt)

c) What is the probability that after servicing ten of these phones, phones of only two of the three types remain to be serviced? (10pt)

d) What is the probability that two phones of each type are among the first six serviced? (5pt)

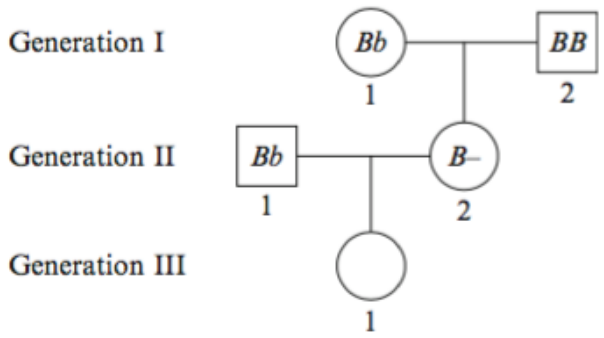
[**4**] Suppose a single gene controls the color of hamsters: black (B) is dominant and brown (b) is recessive. Hence, a hamster will be black unless its genotype is bb.

(Part I) Two hamsters, each with genotype Bb, mate and produce a single offspring. The laws of genetic recombination state that each parent is equally likely to donate either of its two alleles (B or b), so the offspring is equally likely to be any of BB, Bb, bB, or bb (the middle two are genetically equivalent).

a) What is the probability their offspring has black fur? (5pt)

b) Given that their offspring has black fur, what is the probability its genotype is Bb? (5pt)

(Part II) In the figure below, the genotypes of both members of Generation I are known, as is the genotype of the male member of Generation II. We know that hamster II-2 must be black-colored thanks to her father, but suppose that we don't know her genotype exactly (as indicated by B- in the figure).



c) What are the possible genotypes of hamster II-2, and what are the corresponding probabilities? (5pt)

d) If we observe that hamster III-1 has a black coat (and hence at least one B gene), what is the probability her genotype is Bb? If we later discover (through DNA testing on poor little hamster III-1) that her genotype in BB, what is the probability that her mom is also BB? (10pt)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	$P(A\|B) = \frac{P(A \cap B)}{P(B)}$
$P(A \cap B) = P(A\|B)P(B)$	$P(A_k\|B) = \frac{P(B\|A_k)P(A_k)}{\sum_{i=1}^{n} P(B\|A_i)(P(A_i)}$, where $\{A_i\}_{i=1}^{n}$ is a partition.
$P_x^n = \frac{n!}{(n-x)!}$	$C_x^n = \binom{n}{x} = \frac{n!}{x!(n-x)!}$