Chapter 2 - Describing Data: Numerical

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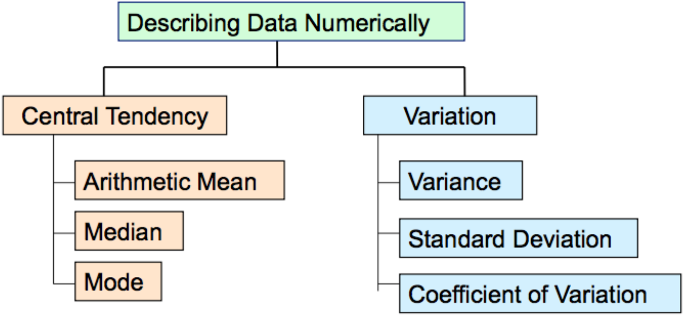
Summer, 2016

Content


Describing Data Numerically
Central Tendency
Arithmetic Mean
Median
Mode
Variation
Variance
Standard Deviation
Coefficient of Variation

Arithmetic Mean

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$$\text{Arith. Mean} = \text{sum of values over number of elements}$$

- Cons: It is affected by extreme values (outliers). Therefore, if there are some extreme values among the sample, then arithmetic mean might not be a good measure of central tendency as the following example demonstrates.

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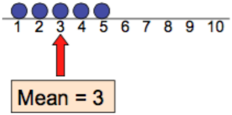
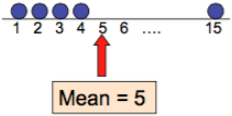
  - Actually, 5 is not even among the values in the sample!





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$$\underbrace{1, 3, 5, 6, 7, 7,}_{6\text{ obs.}} \quad \underbrace{\underbrace{9, 10}_{(9+10)/2}}_{Median=9.5} \quad , \underbrace{10, 10, 11, 12, 13, 13}_{6\text{ obs.}}$$

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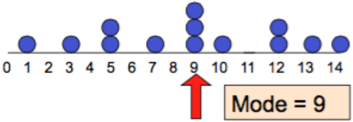
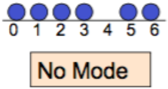
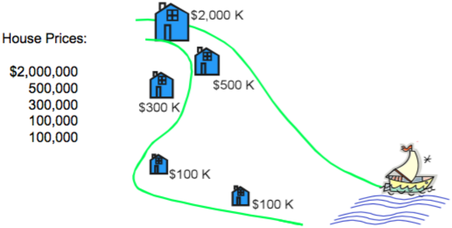




Review Example

Consider the following five houses on a hill by the beach



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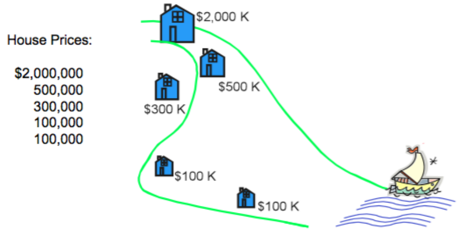

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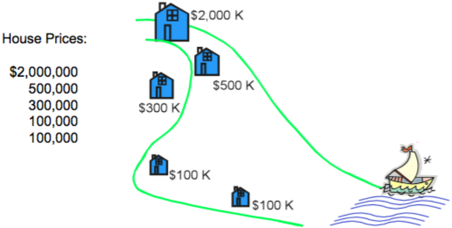

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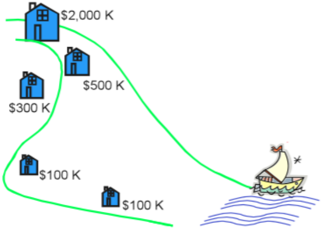

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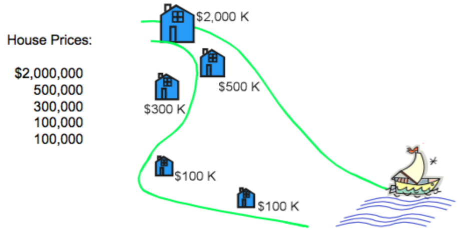

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- But instead we get:
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- So why arithmetic mean doesn't work? Because it doesn't take into account the compound effect of growth!
- Use geometric mean!

Geometric Mean Formula

If growth rate is $r_t\%$ in period t (i.e., the amount from the previous period increases by $r_t\%$), where $t = 1, 2, 3, \ldots, n$, then average growth rate over n years is

$$r = \left[(1 + r_1)(1 + r_2)\ldots(1 + r_n)\right]^{1/n} - 1$$

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- Indeed
  $100 \underbrace{\longrightarrow}_{34.1\%} 134.1 \underbrace{\longrightarrow}_{34.1\%} 180$

What is the average growth rate of sales if sales have grown 25% over the last 5 years?

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$= (1 + r)(1 + r)S$	$= (1 + r_1)(1 + r_2)S$
$= (1 + r)^2 S$	$= \ldots$
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- From these two equations, solving for r yields the geometric mean growth rate formula.

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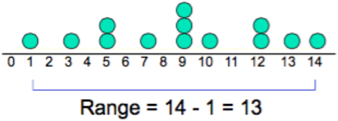

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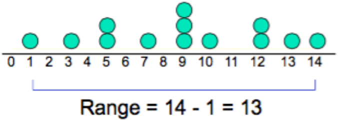

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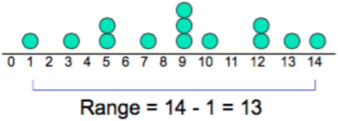

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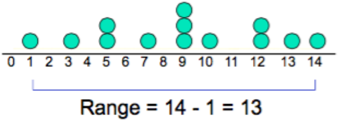

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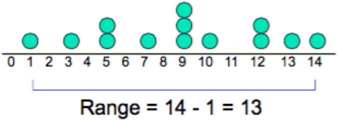

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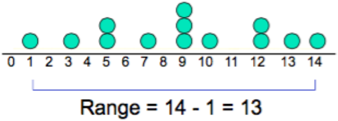

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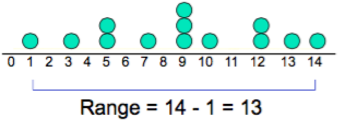

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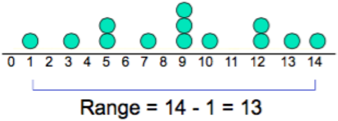

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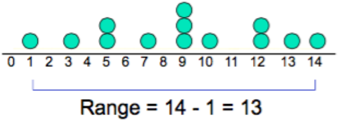

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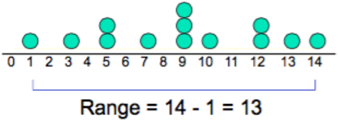

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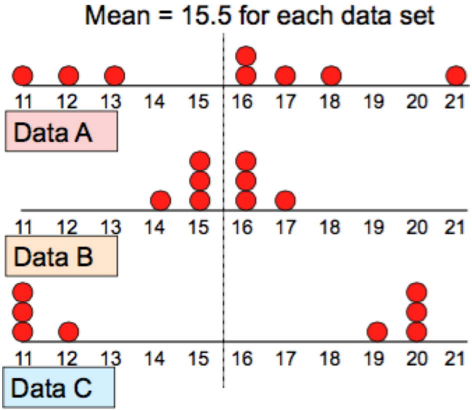

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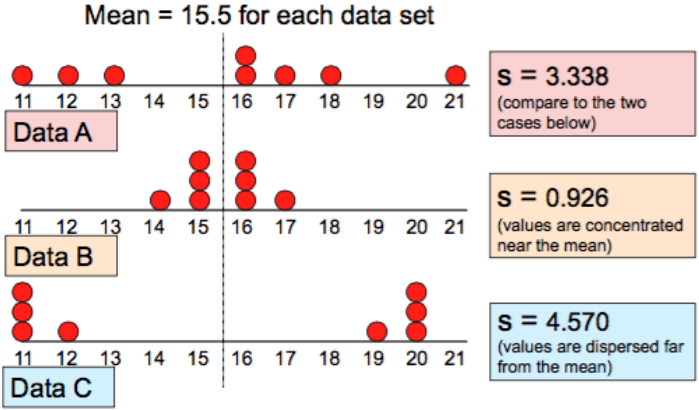

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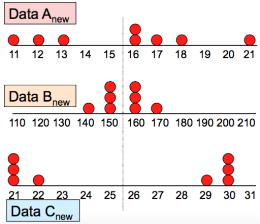

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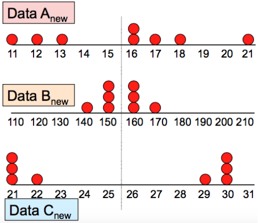

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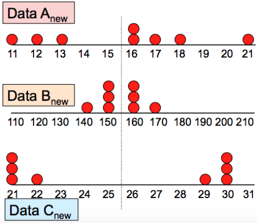

$s_{new} = 3.33$	s = 3.338 (compare to the two cases below)
$s_{new} = 9.2$ (!)	s = 0.926 (values are concentrated near the mean)
$s_{new} = 4.57$	s = 4.570 (values are dispersed far from the mean)

- Only std of data B has changed!
  - **Scaling effects the standard deviation!!!**

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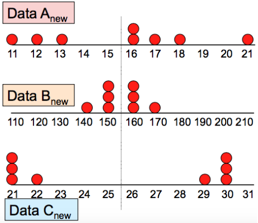

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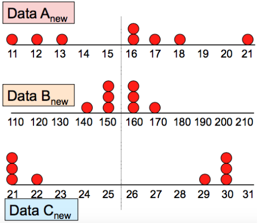

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  - But it has the highest standard deviation!

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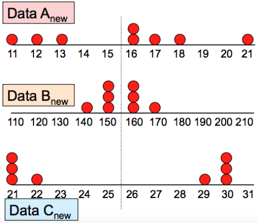

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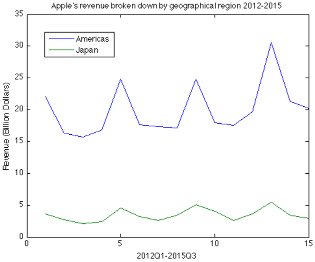

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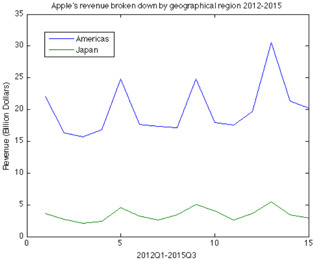

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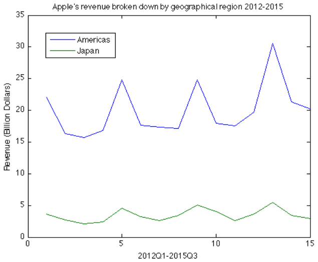

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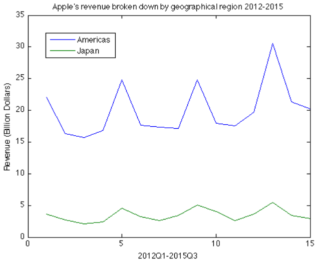

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$$s_{Jap.} = 0.98 \qquad \overline{X}_{Jap.} = 3.5 \qquad \frac{s_{Jap.}}{\overline{X}_{Jap.}} \times 100 = 28\%$$

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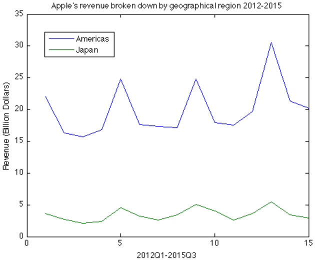

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- Next, we will give a special name to this ratio.

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Sample coefficient of variation:

$$CV = \left(\frac{s}{\overline{X}}\right) \times 100\%$$

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Stock A

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Example:

Stock A

- Average price last year = \$50

Coefficient of Variation

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Example:

Stock A

- Average price last year $= \$50$
- Standard deviation $= \$5$

Coefficient of Variation

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- Average price last year = \$50
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$$\Longrightarrow CV_A = \left(\frac{s_A}{\overline{X}_A}\right) \times 100\%$$

$$= \frac{\$5}{\$50} 100\% = 10\%$$

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Stock B

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Stock B

- Average price last year=\$100

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Stock B

- Average price last year=\$100
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Stock B

- Average price last year=\$100
- Standard deviation = \$5
  $\Longrightarrow CV_B = \left(\frac{s_B}{\overline{X}_B}\right) \times 100\%$
  $= \frac{\$5}{\$100} 100\% = 5\%$

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Stock B

- Average price last year=\$100
- Standard deviation = \$5 $\Longrightarrow CV_B = \left(\frac{s_B}{\overline{X}_B}\right) \times 100\% = \frac{\$5}{\$100} 100\% = 5\%$

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Stock B

- Average price last year=\$100
- Standard deviation = \$5 $\Longrightarrow CV_B = \left(\frac{s_B}{\overline{X}_B}\right) \times 100\% = \frac{\$5}{\$100} 100\% = 5\%$

They have the same standard deviation but Stock A is relatively more volatile.

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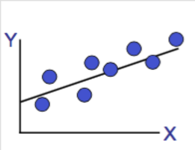

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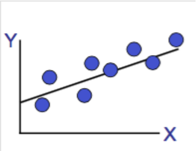

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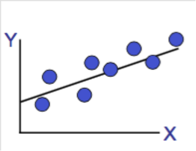

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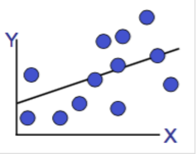

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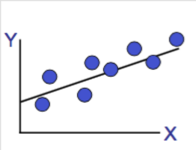

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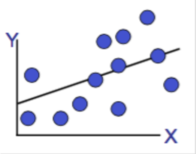

- Which of these two pairs of variables shows a stronger relationship?

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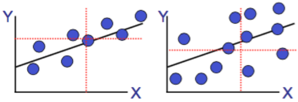

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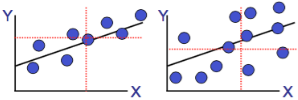

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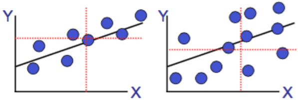

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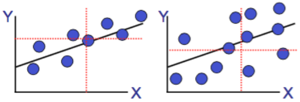

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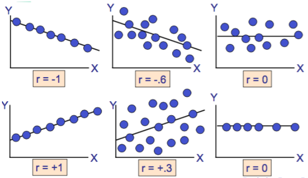

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- Furthermore, suppose that we calculated the sample correlation coefficient as $r = 0.73$.
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