Final Exam

Statistics - NYU, Summer 2016
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[1] A college admissions officer for an MBA program has determined that historically applicants have undergraduate grade point averages that are normally distributed with standard deviation 0.45. Suppose from a random sample of 25 applications from the current year, a statistician obtain the sample mean grade point average as 2.90.

a) Find a 95% confidence interval for the population mean? (5 pt)

b) Based on these sample results, a statistician computes a confidence interval for the population mean that extends from 2.81 to 2.99. Find the confidence level associated with this confidence interval. (7 pt)

c) Suppose that the statistician realizes that each observation in the sample except the median is actually double counted, that is, other than the median each data point mistakenly appears twice in the sample. Also assume that the mean and the median seemed to be equal for this sample. After correcting the data, find a 95% confidence interval for the population mean? (7 pt)

d) Suppose we do not know the population standard deviation, but the grade point averages are still normally distributed. From another random sample of 25 applications from the current year, the sample mean grade point average is calculated as 2.90, and sample standard deviation is found to be 0.40. Find a 95% confidence interval for the population mean? (7 pt)

[2] A simple random sample of 25 filtered cigarettes is obtained, and the tar content of each cigarette is measured. The sample has a mean of 13.2 mg and a standard deviation of 3.7 mg. We are interested in testing the claim that the mean tar content of filtered cigarettes is less than 21.1 mg, which is the mean for unfiltered cigarettes.

a) Formulate the appropriate null and alternative hypotheses. (5 pt)

b) Calculate the appropriate test statistic and conclude the test at $\alpha = 0.05$? (5 pt)

c) Find the p-value? (5 pt)

d) Find the power of the test given that the true population mean tar content is $\mu^* = 17.98$. (10 pt)

e) Construct a 90% confidence interval for the population variance and based on this result briefly discuss whether the population variance can be equal to 8. (7 pt)

[3] Suppose that for a stock analyst A, the length of time to complete an evaluation for a company's earnings forecast is Normally distributed with a standard deviation of 1.7 hours. Among the hundreds of evaluations that have been completed by the analyst A, randomly selected 15 evaluations tells that the average time spent is 6.4 hours, that is $\overline{X} = 6.4$, where X is time spent to complete an evaluation.

a) Construct a 95% confidence interval for the population mean (the true average time spent to complete an evaluation)? (7 pt)

b) Suppose you are told that the probability of getting a sample mean below 6.4 is 0.67, what is the population mean then? (7 pt)

c) Suppose that there is a second analyst, analyst B, who does exactly the same job, but his completion time is normally distributed with a mean of 6.5 hours and a standard deviation of 2.3 hours. For the next 10 cases that they are going to work on, find the probability that the average completion time of analyst A will be at least 1 hour less than the average completion time of analyst B. Assume that their completion times are distributed independently. *(Hint: Use the mean of the population for analyst A that you have found in the previous part and also use the fact that the difference of two normally distributed random variables is again normally distributed)* (7 pt)

[4] Consider big establishments (# of employees > 1000) where the production process is organized as assembly lines mostly. For this type of production organizations, employing workers of similar skills is much more preferred compare to employing workers with dispersed skill levels in order to keep the production cost at minimum. In that sense, these establishments can be classified as either Well-organized or Not-well-organized according to the skill dispersion of the employees. Suppose that skill-dispersion of a company is simply defined as the variance of the skill levels of the employees and the classification is based on the criterion whether the skill-dispersion is below or above 11 units as shown in the following table, which also shows the expected profit per month for each type of establishment.

Skill Variance (σ^2)	Type	Expected Profit ($1000, per month)
$\sigma^2 < 11$	Well-Organized	520
$\sigma^2 > 11$	Not-Well-Organized	330

Now suppose that for a research project you need to carry out certain computations regarding whether FIN-EX, an establishment with 1200 employees, is a well-organized one or not. For that purpose, you obtain a random sample of 15 employees from FIN-EX, and it turns out that the average skill dispersion is 16.7 units in this group.

a) Construct a 90% confidence interval for the true population variance (i.e., overall skill-dispersion in FIN-EX). From this can you conclude whether the factory is Well-Organized or not? (7 pt)

b) Compute the probability that FIN-EX is a well-organized establishment, $P(\sigma^2 < 11) =?$ (7 pt)

c) Compute the expected profit of FIN-EX. (7 pt)